PHILOSOPHICAL STUDIES

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Definitions of "Existence"

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IN A recent study of the logic of existence,[1] Henry S. Leonard analyzed ways of defining "existence" in modern formal logic, i.e., of reconstruing the informal concept "x exists" within the framework of well-known systems of symbolic logic. Leonard distinguishes between *singular existence* as applicable to individuals, and *general existence* as applicable to properties of individuals, and he proposes formal definitions (symbolic renditions) of each of these concepts. I will endeavor to show here that Leonard's proposed definitions of "existence" must be modified, and I shall explore some of the implications of these modifications for the logic of existence-concepts.

As definition of *general existence*—the mode of existence applicable to properties rather than things—Leonard proposes

(1) $\lambda\phi \cdot (\exists x)\phi x$,[2]

i.e., the property ϕ "exists" (has general existence) if there is an individual x which exhibits this property. This definition cannot, I believe, be accepted as it stands. For such properties as "being wiser than Solon," or "being a unicorn," cannot be denied existence qua properties (i.e., general existence), even though no individual object does (or is known to) exhibit them. To qualify a property for general existence it surely suffices that it be *capable of* exemplification, and is not necessary that it be *actually* exemplified. It clearly makes no sense to say "Of course the property P doesn't exist, but it is quite possible that there should be an individual that possesses it."

It appears, therefore, that the modal operator of logical possibility, $\Diamond$, is required adequately to formalize the concept of general existence, and that a modification of (1) by use of this operator is in order. This would be as follows:

(2) $$\lambda\phi \cdot \Diamond (\exists x)\phi x,$$

i.e., the property ϕ "exists" (has general existence) if it *is possible* that there is an individual x which exhibits this property.

In consequence of this definition, general existence would be accorded to expressions which *purport*—i.e., are logically qualified—to characterize individuals, regardless of the (possibly empirical) question of whether there are in fact individuals so characterized. This proposed modification of (1) accords with Leonard's own principle not to permit questions of logic to "wait on the systematic exploration of specific matters of fact," and reinforces his thesis regarding the usefulness of modal logic for the analysis of existence-concepts.

The starting point of Leonard's discussion of *singular existence* is the tentative definition of this concept as

(3) $$\lambda x \cdot (\exists\phi)\phi x,$$

i.e., the individual x "exists" if there is at least one property ϕ which characterizes x. Leonard rejects (3) as definition of "singular existence" on the ground that it is trivialized by the fact that there are some properties—self-identity, $\lambda x \cdot x = x$, for example—which necessarily characterize all (mentionable) objects. He writes, ". . . existence is not implied by necessary, or analytic, predicates. It is, rather, a consequence only of *contingent* truths. . . . Instead of saying that x exists if and only if there is some ϕ such that ϕ of x is true, we must say that x exists if and only if there is some ϕ such that ϕ of x is *contingently* true.[3]

Pursuing this line of thought, Leonard resorts to the modal concept of possibility, defining *singular existence* as

(4) $$\lambda x \cdot (\exists\phi)(\phi x \,\&\, \Diamond \sim\phi x),$$

i.e., the individual x "exists" (has singular existence) if there is at least one *contingent* property ϕ which characterizes x.

I believe that the idea underlying the proposal of (4) as definition of (singular) existence is an important one, but that Leonard's enunciation of this idea is faulty. It is clear from his discussion that he does not intend to deny existence to abstract objects, such as numbers and sets. Yet such objects have no contingent properties: all the properties of 3—e.g., being half of 6—are necessary (demonstrable).[4] Therefore (4) cannot be accepted as an adequate characterization of singular existence.

It is not, however, difficult to rectify this matter, by replacing the requirement for possession of a contingent property by that for possession of a *nontrivial* property, i.e., a property which—unlike self-identity—does not characterize all (mentionable) objects. This modified requirement might appear to be met by changing (4) to

(5) $$\lambda x \cdot (\exists \phi)(\phi x \,\&\, \sim(\psi)\psi x),$$

or equivalently,

(6) $$\lambda x \cdot [(\exists \phi)\phi x \,\&\, (\exists \psi) \sim \psi x],$$

i.e., the individual x "exists" (has singular existence) if there is at least one property ϕ which characterizes x, and at least one property ψ which does *not* characterize x. Now if self-identity, $\lambda x \cdot x = x$, is admitted into the range of our property variables, ϕ, ψ, etc.—as indeed it must be—then the first part of (6) is universally satisfied, and (6) can be reduced to

(7) $$\lambda x \cdot (\exists \phi) \sim \phi x,$$

i.e., the individual x "exists" (has singular existence) if there is at least one property ϕ which x does not possess.

But now again, if self-difference, i.e., $\lambda x \cdot x \neq x$, were admitted into the range of our property variables, (7) would be universally satisfied. This leads us to the recognition that we intend in the present context to confine the application of "property" to those which have *general existence* in the sense considered at the outset, and formulated in (2) above. Modifying (7) so as to render this presupposition explicit, we have

(8) $$\lambda x \cdot (\exists \phi)(\sim \phi x \,\&\, \Diamond (\exists y)\phi y),$$

i.e., the individual x "exists" (has singular existence) if there is at least one property ϕ which does *not* characterize x, but which is such that it is possible that there is some individual y which exhibits this property.[5]

I submit that (8) explicates the concept of singular existence which Leonard has in mind more adequately than does his own proposal (4). On the one hand, (8) does not deny "existence" to abstract objects. On the other hand, it can be seen that "existence" in the sense of (8) is not

trivialized by characterizing all logically possible objects. For let D be a definite description of a possible object $X = (\iota x)Dx$ * which does not in fact exist, so that the set of x's such that Dx is empty (the heaviest unicorn, for example). Now an object x fails to satisfy (8) if

(9) $$\sim(\exists\phi)(\sim\phi x \,\&\, \Diamond(\exists y)\phi y),$$

or equivalently,

(10) $$(\phi)(\Diamond(\exists y)\phi y \supset \phi x),$$

i.e., if any property (having general existence) characterizes x. But anything satisfying D will be characterized by any property ϕ, since $Dx \supset \phi x$ is (trivially) true due to the falsehood of the antecedent.[6] Therefore X is a logically possible object which does not have singular existence in the sense of (8).

I should like, finally, to make use of the foregoing analysis of the concept of *singular existence* to throw light on the question whether "existence" is or is not a *predicate*, that is, a property-denoting term. Let us hypothesize an object X which (like the heaviest unicorn) does not have singular existence in the sense of (8). We thus have

(11) $$\sim(\exists\phi)(\sim\phi X \,\&\, \Diamond(\exists y)\phi y).$$

Let us now assume that existence is a predicate, that is, that $E! =_{Df} \lambda x \cdot (\exists\phi)(\sim\phi x \,\&\, \Diamond(\exists y)\phi y)$ falls within the range of our property variables ϕ, ψ, etc. Now in general we have that

(12) $$\Diamond(\exists y)E!y.$$

Further, we have by (11) that

(13) $$\sim E!X,$$

and therefore,

(14) $$\sim E!X \,\&\, \Diamond(\exists y)E!y.$$

Now under the assumption that E! is within the range of our property-variables, we have by (14) that

(15) $$(\exists\phi)(\sim\phi X \,\&\, \Diamond(\exists y)\phi y).$$

But this contradicts the hypothesis (11). It follows that existence ("singular existence") is not a predicate in the precise sense that the characteristic (8) does not fall within the range of the property-variables occurring therein.

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NOTES

[1] "The Logic of Existence," *Philosophical Studies*, 7:49–64 (1956).

* EDITORS' NOTE: Limitations of the linotype font have made it necessary to use the regular instead of the inverted iota.

[2] I use the symbolism '$\lambda a \cdot F(a)$' to denominate that characteristic exhibited by a if and only if a satisfies the condition F.

[3] *Op. cit.*, p. 57.

[4] Leonard appears to hold (p. 58, *ibid.*) that every object has a contingent property, but his discussion here overlooks abstract objects. His proof that any individual x "exists" in the sense of (4) is in error on precisely this point.

[5] By substitution of '$\sim\phi$' for 'ϕ,' (8) could be reformulated as $\lambda x \cdot (\exists \phi)(\phi x \,\&\, \Diamond (\exists y) \sim\phi y)$. In comparison with (4) we see that the only point of difference is that the individual which possibly does not exhibit ϕ need not be x itself.

[6] It is assumed that "ι" is so defined that $(y)(\psi y \supset \phi y) \supset \phi(\iota x)\psi x$.

This assumption is false in *Principia Mathematica* in view of the particular definition of "ι" used in that work (see *14). But other, alternative definitions of "ι" are available for which this assumption is satisfied. One such is the contextual definition: $F(\iota x)\phi x = {}_{Df}(y)(y \in$ the x's such that $\phi x \supset Fy)$. With this definition, the assumption is readily seen to be satisfied. Here it is easily shown that $(\iota x)\phi x$ is specified as follows: (1) if there is *exactly one* individual for which ϕ holds, say a, then $(\iota x)\phi x = a$ (this can be seen by letting $F\{X\}$ be $X = z$ in this case), (2) if there is *no* individual for which ϕ holds, then $(\iota x)\phi x = \Lambda$ (let $F\{X\}$ be $X \subset z$), so that in this case $(\iota x)\phi x$ is the null class, and (3) if there are *several* individuals for which ϕ holds, then $(\iota x)\phi x$ is some random or representative element of the x's such that ϕx. The definition under consideration is thus very close to that given by Frege in *Grundgesetze der Arithmetik* (vol. 1, Jena, 1873, p. 19), who defines $(\iota x)\phi x$ to be a, Λ, or (all of) the x's such that ϕx, respectively in these three cases.